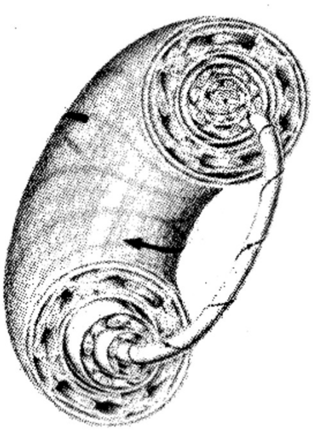

Figure 2.4 Nested tori for a slightly perturbed integrable system. Note the hierarchy of elliptic orbits interspersed with chaotic regions. A magnification of this heirarchy would show the same pattern repeated on a smaller scale and so on, *ad infinitum*. (Reproduced from [Ab78].)

Large deviations from integrability must be investigated numerically. One convenient case for doing so is the potential

$$V(x,y)=\frac{1}{2}\left(x^2+y^2\right)+x^2y-\frac{1}{3}y^3, \qquad (2.35)$$

which was originally introduced by Henon and Heiles in the study of stellar orbits through a galaxy [He64]. This potential can be thought of as a perturbed harmonic oscillator potential (a small constant multiplying the cubic terms can be absorbed through a rescaling of the coordinates and energy, so that the magnitude of the energy becomes a measure of the deviation from integrability) and has the three-fold symmetry shown in Figure 2.5. The potential is zero at the origin and becomes unbounded for large values of the coordinates. However, for energies less than 1/6, the trajectories remain confined within the equilateral triangle shown.

The FORTRAN program for Example 2, whose source code is contained in Appendix B and in the file EXMPL2.FOR on the *Computational Physics* diskette, constructs surfaces of section for the Henon-Heiles potential. The method used is to integrate the equations of motion (2.31) using the fourth-order Runge-Kutta algorithm (2.25). Initial conditions are specified by putting $x=0$ and by giving the energy, y, and p_y; p_x is then is fixed by energy conservation. As the integration proceeds, the (x,y) trajectory and the (y,p_y) surface of section are displayed. Points on the latter are calculated by watching for a time step during which x changes sign. When this happens, the precise location of the point on the surface of section plot is determined by switching to x as the independent variable, so that the equations of motion (2.31) become

$$\frac{dx}{dx}=1\ ,\ \frac{dy}{dx}=\frac{1}{p_x}p_y;$$